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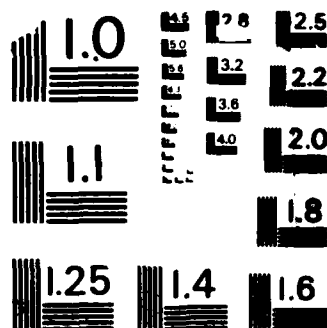
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**STOCHASTIC DYNAMIC SYSTEMS WITH MULTIPLE DECISION MAKERS
AND PARAMETRIC UNCERTAINTIES**

INTERIM REPORT TO AFOSR

for the period

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December 6, 1985

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1. MULTIPLE DECISION-MAKER PROBLEMS WITH UNKNOWN PARAMETERS

The problem of strategic decision making in complex systems which involve multiple decision makers (DM's), multiple objectives, and incomplete information arises frequently in the military context, and in particular in the Command, Control, and Communications (C³) systems field. As compared with single DM problems, the analysis of multiple DM problems requires different approaches and techniques, and furthermore certain standard features and properties we usually ascribe to single DM problems do not generally extend naturally to multiple decision making. For example, while, in single DM problems, optimization (minimization or maximization) of a single objective functional would, in general, lead to a satisfactory decision policy (the so-called *optimal policy*), when the decision problem involves multiple DM's and multiple objectives a plethora of possibilities emerge as to the criterion which leads to a "satisfying" set of policies. Depending on the number of DM's, their underlying goals, and the presence or absence of dominance in the decision making process, we may have *team-optimal*, *person-by-person optimal*, *Pareto optimal*, *Nash equilibrium*, *Stackelberg* (leader-follower) *equilibrium* concepts, and several variants of combinations of these in case of more than two DM's. Each of these in general leads to a different outcome which is also a variant of the information structure of the problem (i.e., what each DM knows a priori, what information he acquires during the evolution of the decision process, what information exchange links are allowable, and what information transmission capability each DM is vested with). The significance of information structure in multiple DM problems also manifests itself in the derivation of multimodel strategies: Model simplification through singular perturbations or aggregation is not a well-posed procedure

unless there is some kind of a matching between the information structures of the original problem and the simplified version--no such inconsistencies arise, however, in single decision-maker problems.

Recent years have witnessed considerable advances in our understanding of equilibrium solutions of deterministic and stochastic multi-person decision problems, and in particular as regards the Stackelberg equilibrium solution. A class of such Stackelberg problems which were long thought to be extremely challenging have recently been solved using indirect methods, for both deterministic and stochastic systems. In some cases it has been shown that the Stackelberg equilibrium strategy for the leader forces the DM's at lower levels of hierarchy to a team behavior, jointly optimizing the leader's performance index, even though they may each have different goals and performance indices. In other cases, tight performance bounds have been obtained on the leader's cost function, which are achievable by implementable policies.

A large majority of this work on multiple DM problems pertains to either deterministic systems or to systems with uncertain elements which have a complete probabilistic description--this a priori information being known by all the DM's (the latter class of problems are also known as stochastic dynamic games). Hence, even though some decentralization of dynamically acquired information has been allowed for in the general formulation of dynamic games, it has been a common assumption to endow every DM with the common (centralized) a priori information regarding the complete statistical description of the "primitive" random variables. This, however, is not always a realistic assumption, in particular when the decision problem

involves multiple DM's and multiple objectives. A more realistic formulation, in most cases, would involve a number of uncertain parameters which are either not stochastic or they are stochastic but their complete statistical description is not known by all the DM's.

The presence of unknown (or uncertain) parameters could affect the general problem formulation in basically three different ways:

- i) *Through the objective functions.* Here, the objective function of the i 'th DM may not be known completely by the j 'th DM ($j \neq i$), with the uncertainty characterized by a number of parameters whose values are unknown to the j 'th DM.
- ii) *Through the system response.* The evolution of the decision process may depend on a number of parameters whose values are unknown to some or all DM's. [This type of uncertainty is also applicable to stochastic team problems.]
- iii) *Through the measurements made by the DM's.* Here either the observation scheme or the statistics of some of the variables in the measurement process of a DM (or both) may not be known to some other DM, with the uncertainty again being parameterized. [As in ii) this type of uncertainty is also applicable to stochastic team problems.]

Multiple DM problems with the types of uncertainties as described above can be treated by adopting essentially one of the following three approaches:

- a) *Robustness or Minimum Sensitivity Approach.* Here one assumes some nominal values for the unknown parameters, determines a corresponding nominal performance for the system, and designs decision policies which would lead to minimum performance degradation should the parameters vary around their nominal values. The resulting decision policies are called *minimum sensitivity strategies*, and they are robust in a certain neighborhood of the nominal values.
- b) *Learning Schemes.* In this approach no nominal values for the unknown parameters will be available, but some a priori statistics may be attached to these parameters by the DM's, which will be updated in a decentralized manner as new dynamic information is acquired. This is akin to some of the methodologies developed earlier for control problems with unknown parameters (such as identification, parameter estimation, and adaptive control--which are still active research areas), which are, however, not applicable to multiple DM problems because the rather intricate interactions of multiple DM's render any central learning scheme infeasible.
- c) *Minimax Approach.* Here no nominal values are available for the unknown parameters, but they are known to belong to some pre-specified sets. Then, the objective is to design strategies which would carry optimality or equilibrium property under worst possible values of the parameters on these sets. Such an approach entails a pessimistic design philosophy, and is applicable mostly to decision problems with a common objective functional (i.e., team problems). In multi-objective problems, the minimax philosophy is somewhat ambiguous at the current stage of

development, since what may seem to be a worst-case design for one objective functional may seem to lose this property when tested against a different objective functional. However, if different objective functionals are affected by different sets of unknown parameters, this approach would still be applicable, and further research would definitely be needed to study ramifications of such a line of approach in these problems.

We should point out that a combination of any two or all three of the above approaches would also constitute a viable approach to multi-person decision problems with unknown parameters, which should be studied in proper contexts once the rudiments of a theory for each one separately is laid down.

2. RESEARCH ACCOMPLISHMENTS

In our proposal, we recognized the fact that the class of multiple DM problems with uncertain parameters, as described above, are still in their infancy, in particular under the "Learning Scheme" approach. In view of this, we proposed to initiate original fundamental research to make theoretical advances in this field and to design implementable decision policies which carry both the learning and command capabilities. In doing this, we proposed to adopt the general framework of deterministic and stochastic dynamic games, and to study these problems under three types of uncertainty discussed in Section 1, and under different solution concepts such as team-optimal, Nash equilibrium, and Leader-Follower (hierarchical).

During the first year of this project, we have addressed several challenging issues in this context, and have made important advances. We briefly outline some of these new results in the sequel; full details can be found in the references listed in Section 3.

In the first group of papers, listed in Section 3 as [P1]-[P3], we have adopted the first (i.e., minimum sensitivity) approach for a class of decision problems which displayed the first type of uncertainty, viz. the case of one of the DMs' cost function depending on a number of parameters whose precise values are known by him but not by other(s). In [P1], we have presented a general mathematical formulation and a method of solution for stochastic incentive decision problems, using concepts and tools of dynamic game theory. As special cases of the general formulation we have considered four different classes of problems which differ in the information available

to the DM's, their objectives, and the numbers of DM's at different levels of hierarchy. The fourth class we considered can be viewed as an "exact model matching" problem akin to the one arising in nonlinear control. In the paper, an explicit incentive policy has been obtained for the DM occupying the higher level in the hierarchy, which, besides solving the exact matching problem, carried very appealing minimum sensitivity properties. These features have also been demonstrated in [P1] in the context of a numerical example. The other two papers, [P2] and [P3], extend these results to more general models, with the former devoted to decision problems defined on finite dimensional spaces, and the latter dealing with nominally team problems (i.e., decision problems with a common objective functional) defined on infinite dimensional spaces. Thus, the formulation of [P3] covers also stochastic control problems defined in the continuous time, with multiple decentralized controllers, and allowing for parametric uncertainty in the overall modeling from the viewpoint of some of the stations.

The fourth paper listed in Section 3, [P4], addresses a different class of problems, wherein the uncertainty is of the second and third types (see Section 1) and the general approach is the "learning scheme"; here, all three solution concepts, viz. Nash, hierarchical, and Pareto-optimal, are employed. The discussion of [P4] includes both finite and infinite-state two-person decision models, with the uncertainties being in the statistical description of the random variables appearing in the system dynamics, and the measurements of the two DM's, each DM developing a different prior on these random variables. The paper develops different recursive schemes which involve "learning" in the policy space and lead to policies that converge

to the equilibrium under different stipulations on the information structure of the problem. We have also analyzed the robustness and sensitivity of team optimal solutions to deviations in the perceptions of the DM's from a common stochastic model, and have shown that adoption of the Nash equilibrium solution leads to well-posed models, whereas the other two solution concepts lead to bifurcation once deviated from the nominal model. An important by-product of this theoretical analysis is a recursive relationship which leads to the optimal solution of a quadratic stochastic team problem with decentralized information, in which the underlying statistics are not Gaussian. There seems to be considerable potential in this approach to decentralized stochastic control (team) problems with non-Gaussian statistics, leading to a converging numerical scheme for the derivation of optimal policies--thus solving an important class of stochastic optimization problems which have remained unsolved until today.

The fifth paper, [P5], deals with a fundamental problem in dynamic game theory, which is development of a theory of noncooperative equilibrium for decision problems whose dynamics are described by higher (than one) order difference equations. Even though such extensions are trivial in the case of deterministic optimal control problems (simply increase the dimension of the state space by introducing new state variables), this is quite a nontrivial task in game problems. The paper first discusses the reasons behind the intricacies involved, and then presents a general procedure to obtain informationally unique noncooperative Nash equilibria in the presence of random disturbances, with the theoretical result illustrated by a numerical example.

The sixth paper, [P6], addresses a decentralized large scale decision (team) problem with N DM's, and introduces a novel procedure to obtain

suboptimal policies with appealing features. It utilizes the method of chained aggregation to decompose the overall team problem into $(N+1)$ subproblems: one low order team problem with a centralized information structure and N decentralized optimal control problems. Accordingly, the control of each DM is decomposed into three components: a decoupling control which induces aggregation, a local control which controls the subsystem dynamics, and an aggregate control which controls the dynamics of the interconnection variables. The paper also establishes the robustness of this composite control with respect to perturbations in the system dynamics and the cost functional.

3. PUBLICATIONS SUPPORTED BY THE GRANT

- [P1] Tamer Başar, "Dynamic Games and Incentives," in *Systems and Optimization, Lecture Notes in Control and Information Sciences*, A. Bagchi and H. T. Jongen (eds.), Springer-Verlag, Vol. 66, 1985, pp. 1-13.
- [P2] Derya H. Cansever and Tamer Başar, "Optimum/Near-Optimum Incentive Policies for Stochastic Decision Problems in the Presence of Parametric Uncertainty," *Automatica*, Vol. 24, No. 5, September 1985.
- [P3] Derya H. Cansever and Tamer Başar, "Near-Optimum Incentive Policies in Stochastic Team Problems with Discrepancies in Goal Perceptions," *Proc. 23rd IEEE Conf. on Decision and Control*, 1984, pp. 1188-1193.
- [P4] Tamer Başar, "Stochastic Multimodeling for Teams in a Game-Theoretic Framework," in *Optimal Control Theory and Economic Analysis 2*, G. Feichtinger (ed.), 1985, pp. 529-548.
- [P5] Tamer Başar, "Informational Uniqueness of Closed-Loop Nash Equilibria for a Class of Nonstandard Dynamic Games," in *J. Optimization Theory and Applications*, Vol. 46, No. 4, August 1985, pp. 409-419.
- [P6] V. R. Saksena and J. B. Cruz, Jr., "An Approach to Decentralized Control of Large Scale Systems Using Aggregation Methods," *IEEE Transactions on Automatic Control*, Vol. AC-30, No. 9, September 1985, pp. 912-914.

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